

Settling Multiple Debts: Lesson Plan

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Introduction

The teacher needs to explain the setting.

A group of friends lend each other money throughout the year. They carefully record each transaction. When Alice lends 10 currency units to Bob, this is recorded as

$$\text{Alice} \xrightarrow{10} \text{Bob}$$

At the end of the year they wish to settle their debts. How should they transfer money so as to settle all debts?

Give them a simple 3-person case to try:

$$\begin{array}{l} \text{Alice} \xrightarrow{10} \text{Bob} \\ \text{Alice} \xrightarrow{20} \text{Carol} \\ \text{Bob} \xrightarrow{15} \text{Carol} \\ \text{Carol} \xrightarrow{25} \text{Alice} \end{array}$$

What Issues Can/Should Arise?

When time permits, it is best to let the students do the discovering. First, let them work on some problems individually, then have them compare their results. Discuss some issues that arise.

1. Look at simpler cases: 2-person

2. Formulate concrete problems that are equivalent: banks settling their mutual debts via money trucks, transporting sand from depots to construction sites, shipping ores from mines to processing plants, ...
3. The notion of a person's *balance*, and how to calculate them
4. How to state the problem?
5. What input is given? A list of transfers
6. What output is to be produced? For small groups, it is enough to produce the balances (especially when using a central account or table to settle). More explicit: specify set of transactions with source, destination, and amount
7. What settling schemes are acceptable? For example, using a central account?
8. Is one way of settling preferred over another? Why?
9. Efficiency: total amount transferred, number of transfers
10. How to reason about minimality of total amount transferred? Show lower bound, and achieve it.
11. Upper and lower bounds on minimum number of transfers
12. Notations: 3-column table (as in the example), matrix, graph
13. Separation of concerns: deal with one issue at a time
14. Algorithm to produce settlement with minimum total amount transferred
15. Does the algorithm produce a correct settlement?
16. How efficient is the algorithm? Run time, memory usage;
17. How good is the settlement produced? Minimum total amount transferred? How about the number of transfers?